

# Stochastic Gravity: Beyond Semiclassical Gravity

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The back-reaction of a classical gravitational field interacting with quantum matter fields is described by the semiclassical Einstein equation, which has the expectation value of the quantum matter fields stress tensor as a source. The semiclassical theory may be obtained from the quantum field theory of gravity interacting with  $N$  matter fields in the large  $N$  limit. This theory breaks down when the fields quantum fluctuations are important. Stochastic gravity goes beyond the semiclassical limit and allows for a systematic and self-consistent description of the metric fluctuations induced by these quantum fluctuations. The correlation functions of the metric fluctuations obtained in stochastic gravity reproduce the correlation functions in the quantum theory to leading order in an  $1/N$  expansion. Two main applications of stochastic gravity are discussed. The first, in cosmology, to obtain the spectrum of primordial metric perturbations induced by the inflaton fluctuations, even beyond the linear approximation. The second, in black hole physics, to study the fluctuations of the horizon of an evaporating black hole.

PACS numbers: 04.62.+v, 03.65.Sq, 05.40.-a

## I. INTRODUCTION

There are two aspects to semiclassical gravity. On the one hand we have quantum field theory in a curved spacetime which is now a well understood and well defined theory both for free fields [1] and interacting fields [2]. In this theory the gravitational field is the classical field of general relativity, the metric of the spacetime, and the quantum fields are test fields which propagate in such a spacetime. Since the spacetime is now dynamical it is not always possible to define a physically meaningful vacuum state for the quantum field and when this is possible in some “initial” times it is usually unstable, in the sense that it may differ from the vacuum state at latter times, and spontaneous creation of particles occurs. Applications of this in cosmology, such as particle production in expanding Friedmann-Robertson-Walker models [3, 4], and black hole physics, such as Hawking radiation [5, 6], are well known. This is one aspect of the interaction of gravity with quantum matter fields

The second aspect of this interaction is the back-reaction of the quantum fields on the spacetime. Since the gravitational field couples to the stress tensor of matter fields, the key object here is the expectation value in a given quantum state of the stress tensor of the quantum field, which is a classical observable. However, since this object is quadratic in the field operator, which is only well defined as a distribution on spacetime, it involves ill defined quantities which translate into ultraviolet divergences. To be able to define a physically meaningful quantity a regularization and a renormalization procedure is required. The ultraviolet divergences associated to the expectation value of the stress tensor are also present in Minkowski spacetime, but in a curved background the renormalization procedure is more sophisticated since it needs to preserve general covariance. A regularization procedure which is specially adapted to the curved background is the so called point-splitting method [7, 8, 9]. The final expectation value of the stress tensor using point splitting or any other reasonable regularization technique such as dimensional regularization, is essentially unique, modulo some terms which depend on the spacetime curvature and which are independent of the quantum state. This uniqueness was proved by Wald [10, 11, 12] who investigated the criteria that a physically meaningful expectation value of the stress tensor ought to satisfy.

The back-reaction is formulated in terms of the semiclassical Einstein equations. These are Einstein equations which have the expectation value in some quantum state of the stress tensor as a matter source. The back-reaction problem was investigated in cosmology, in particular to see whether cosmological anisotropies could be damped by back-reaction [13, 14]. This was an earlier attempt [15, 16] previous to inflation to explain why the universe is so isotropic at present.

A useful approach to the back-reaction problem is to use the effective action methods [17, 18, 19] that are familiar in quantum field theory. These methods were of great help in the study of cosmological anisotropies since they allowed the introduction of familiar perturbative treatments into the subject. The most common effective action method,

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however, led to equations of motion which were not real because they were tailored to compute transition elements of quantum operators rather than expectation values. Fortunately the appropriate technique had already been developed by Schwinger and Keldysh [20, 21, 22] in the so called Closed Time Path (CTP) or “in-in” effective action method, and was soon adapted to the gravitational context [23, 24, 25, 26]. These techniques were then applied to different problems in the cosmological context including the effects of arbitrary perturbations on homogeneous backgrounds [27]. As a result one was able to deduce the semiclassical Einstein equations by the CTP functional method: treating the matter fields as fully quantum fields and the gravitational field as a classical field.

The semiclassical Einstein equations have limitations even outside the Planck scales: when the fluctuations around the expectation value of the stress tensor of the matter fields are large the semiclassical equations should break down [28, 29, 30]. One expects, in fact, that a better approximation would describe the gravitational field in a probabilistic way. In other words, that the semiclassical equations should be substituted by some Langevin-type equations with a stochastic source that describes the quantum fluctuations. A significant step in this direction was made by Hu [31] who proposed to view the back-reaction problem in the light of the open quantum system paradigm, where the quantum fields play the role of the “environment” and the gravitational field plays the role of the “system”. Following this proposal a systematic study of the connection between semiclassical gravity and open quantum systems resulted in the development of a framework where different semiclassical Einstein-Langevin equations were derived [32, 33, 34, 35]. The key technical factor to most of these results was the use of the influence functional method of Feynman and Vernon [36, 37] for the description of the system-environment interaction when only the state of the system is of interest. The CTP method for open systems involves, in fact, the influence functional.

However although several Einstein-Langevin equations were derived, these were always partial and related to some particular cosmological situation. On the other hand, the results were somewhat formal and some concern could be raised on the physical reality of the solutions of the stochastic equations for the gravitational field. This is related to the issue of the environment induced quantum to classical transition. In fact, for the existence of a semiclassical regime for the dynamics of the system one needs two requirements, in the language of the consistent histories formulation of quantum mechanics [38, 39]. The first is decoherence, which guarantees that probabilities can be consistently assigned to histories describing the evolution of the system, and the second is that these probabilities should peak near histories which correspond to solutions of classical equations of motion. The effect of the environment is crucial on the one hand to provide decoherence [40] and on the other hand to produce both dissipation and noise to the system through back-reaction, thus inducing a semiclassical stochastic dynamics on the system. As shown by Gell-Mann and Hartle [41] in an open quantum system stochastic semiclassical equations are obtained after a coarse graining of the environmental degrees of freedom and a further coarse graining in the system variables. That this mechanism could also work for decoherence and classicalization of the metric field was not so clear lacking a full quantum description of the gravitational field, and the analogy could be made only formally [42].

An alternative axiomatic approach to the Einstein-Langevin equations which was independent of the open system analogy was introduced: it was based on the formulation of a general and consistent dynamical set of equations for a perturbative correction to semiclassical gravity able to account for the lowest order quantum stress tensor fluctuations of matter fields [43]. It was later shown that these same equations could be derived, in this general case, from the influence functional of Feynman and Vernon in which, the gravitational field is treated as a classical field and the quantum fields are quantized, the first being, in fact, the “system” and the second the “environment” [44]. Also, inspired by results in some simple open quantum systems [45] and results of stochastic gravity in Minkowski spacetime [46], the concern on the reality of the stochastic solutions was latter clarified. It was realized that the correlation functions of the metric fluctuations obtained in stochastic gravity reproduce the correlation functions in the quantum theory of gravity interacting with  $N$  quantum fields to leading order in an  $1/N$  expansion [47, 48]. Thus, stochastic gravity may be understood as a powerful and useful framework to study quantum metric fluctuations.

Here we review the development of stochastic gravity and some of its applications. In section II a brief sketch of semiclassical gravity is given. In section III the axiomatic approach to stochastic gravity is discussed by introducing the Einstein-Langevin equations. In section IV to illustrate the relation between the semiclassical, stochastic and quantum theories, a simplified model of scalar gravity interacting with  $N$  scalar fields is considered. In section V we review an important application of stochastic gravity in cosmology. It concerns the computation of the two-point correlations of the metric perturbations induced by the fluctuation in the stress tensor of the inflaton field during inflation. The results to linear order agree with the standard results but the present method, in which the matter fields are treated exactly, may go beyond the usual approaches where the inflaton fluctuations are treated at linear level only. In section VI we deal with another important application in black hole physics: the study of the fluctuations near the horizon of an evaporating black hole. This subject is still under consideration and we only sketch some of the recent results. Finally, in section VII we summarize our result and briefly discuss other applications. We should mention that two reviews of stochastic gravity and its applications are now available [49, 50].

## II. SEMICLASSICAL GRAVITY

Semiclassical gravity is a theory which describes the interaction of the gravitational field assumed to be a classical field with matter fields which are quantum. It is supposed to be some limit of the still unknown quantum theory describing the interaction of gravity with other fields. Due to the lack of the full quantum theory, the semiclassical limit cannot be rigorously derived. However, it can be formally derived in several ways. One of them is the leading-order  $1/N$  approximation of quantum gravity [51], where  $N$  is the number of independent free quantum fields which interact with gravity only. In this limit, after path integration one arrives at a theory in which the gravitational field can be treated as a c-number (i.e. is quantized at tree level) and the quantum fields are fully quantized. If we call  $g_{ab}$  the metric tensor and  $\hat{\phi}$  the scalar field (for simplicity we consider just one scalar field) one arrives at the semiclassical Einstein equation as the dynamical equation for the metric  $g_{ab}$ :

$$G_{ab}[g] = \kappa \langle \hat{T}_{ab}[g] \rangle_{ren}, \quad (1)$$

where  $\kappa = 8\pi G = 8\pi/m_P^2$ ,  $G$  is Newton's gravitational constant and  $m_P$  is the Planck mass,  $\hat{T}_{ab} = T_{ab}[\hat{\phi}^2]$  is the stress tensor operator which is quadratic in the field operator  $\hat{\phi}$ . This operator, being the product of distribution valued operators, is ill defined and needs to be regularized and renormalized, the subscript *ren* means that the operator has been renormalized. The angle brackets on the right hand side mean that the expectation value of the stress tensor operator is computed in some quantum state, say  $|\psi\rangle$ , compatible with the geometry described by the metric  $g_{ab}$ . On the left hand side  $G_{ab}[g]$  stands for the Einstein tensor for the metric  $g_{ab}$  together with the cosmological constant term and other terms quadratic in the curvature which are generally needed to renormalize the stress tensor operator. The quantum field operator  $\hat{\phi}$  propagates in the background defined by the metric  $g$ , it thus satisfies a Klein-Gordon equation,

$$(\nabla_g^2 - m^2 - \xi R[g]) \hat{\phi} = 0, \quad (2)$$

where  $\nabla_g^2$  stands for the D'Alembert operator in the background  $g_{ab}$ ,  $\xi$  is a dimensionless coupling parameter ( $\xi = 0$  is the minimal coupling and  $\xi = 1/6$  is the conformal coupling) and  $R$  is the Ricci scalar for the background metric. Equation (1) is the semiclassical Einstein equation, it is the dynamical equation for the metric tensor  $g_{ab}$  and describes the back-reaction of the quantum matter fields on the geometry. A solution of semiclassical gravity consists of the set  $(g_{ab}, \hat{\phi}, |\psi\rangle)$  where  $g_{ab}$  is a solution of (1),  $\hat{\phi}$  is a solution of (2) and  $|\psi\rangle$  is the quantum state in which the expectation value of the stress tensor in equation (1) is computed.

This theory is in some sense unique as a theory where the gravitational field is classical. In fact, the (classical) gravitational field interacts with matter fields through the stress tensor, and the only reasonable c-number stress tensor that one may construct [10] with the operator  $\hat{T}_{ab}$  is just the right hand side of (1), modulo the curvature terms needed for renormalization. However the scope and limits of the theory are not so well understood as a consequence of the lack of the full quantum theory. It is assumed that the semiclassical theory should break down at Planck scales, which is when simple order of magnitude estimates suggest that the quantum effects of gravity cannot be ignored: the gravitational energy of a quantum fluctuation of energy in a Planck size region, determined by the Heisenberg uncertainty principle, is of the same order of magnitude as the energy fluctuation.

There is also another situation when the semiclassical theory should break down, namely, when the fluctuations of the stress tensor are large. This has been emphasized by Ford and collaborators. It is less clear how to quantify what a large fluctuation here means and some criteria have been proposed [30, 52, 53]. Generally this depends on the quantum state and may be illustrated by the example used in ref. [29] as follows.

Let us assume a quantum state formed by an isolated system which consists of a superposition with equal amplitude of one configuration with mass  $M_1$  and another with mass  $M_2$ . Semiclassical theory as described in (1) predicts that the gravitational field of this system is produced by the average mass  $(M_1 + M_2)/2$ , that is a test particle will move on the background spacetime produced by such a source. However one would expect that if we send a succession of test particles to probe the gravitational field of the above system, half of the time they would react to the field of a mass  $M_1$  and the other half to the field of a mass  $M_2$ . If the two masses differ substantially the two predictions are clearly different, note that the fluctuations in mass of the quantum state is of the order of  $(M_1 - M_2)^2$ . Although the previous example is suggestive a word of caution should be said in order not to take it too literally. In fact, if the previous masses are macroscopic the quantum system decoheres very quickly [40] and instead of a pure quantum state it is described by a density matrix which diagonalizes in a certain pointer basis. For observables associated to this pointer basis the matrix density description is equivalent to that provided by a statistical ensemble. In any case, however, from the point of view of the test particles the predictions differ from that of the semiclassical theory.

### III. STOCHASTIC GRAVITY

The purpose of semiclassical stochastic gravity, or stochastic gravity for short, is to be able to deal with the situation of the previous example in which the predictions of the semiclassical theory may be too rough. Consequently, our first point is to characterize the quantum fluctuations of the stress tensor.

The physical observable that measures these fluctuations is related to the two-point stress tensor correlations. Let us consider the tensor operator  $\hat{t}_{ab} \equiv \hat{T}_{ab} - \langle \hat{T}_{ab} \rangle \hat{I}$ , where  $\hat{I}$  is the identity operator, and introduce the *noise kernel* as the four index bi-tensor defined as the expectation value of the anticommutator of the operator  $\hat{t}_{ab}$ :

$$N_{abcd}(x, y) = \frac{1}{2} \langle \{ \hat{t}_{ab}(x), \hat{t}_{cd}(y) \} \rangle. \quad (3)$$

This expectation value is taken in the background metric  $g_{ab}$  which we assume to be a solution of the semiclassical equation (1). An important property of the symmetric bi-tensor,  $N_{abcd}(x, y) = N_{cdab}(y, x)$ , is that it is finite because the tensor operator  $\hat{t}_{ab}$  is finite since the ultraviolet divergences of  $\hat{T}_{ab}$  are cancelled by the subtraction of  $\langle \hat{T}_{ab} \rangle$ . Since the operator  $\hat{T}_{ab}$  is selfadjoint  $N_{abcd}(x, y)$ , which is the expectation value of an anticommutator, is real and positive semi-definite. This last property allows for the introduction of a classical Gaussian stochastic tensor  $\xi_{ab}$  defined by

$$\langle \xi_{ab}(x) \rangle_s = 0, \quad \langle \xi_{ab}(x) \xi_{cd}(y) \rangle_s = N_{abcd}(x, y). \quad (4)$$

This stochastic tensor is symmetric  $\xi_{ab} = \xi_{ba}$  and divergenceless,  $\nabla^a \xi_{ab} = 0$ , as a consequence of the fact that the stress tensor operator is divergenceless. The subscript  $s$  means that the expectation value is just a classical stochastic average. Note that we assume that  $\xi_{ab}$  is Gaussian for simplicity, in order to include the main effect. The idea now is simple, we want to modify the semiclassical Einstein equation (1) by introducing a linear correction to the metric tensor  $g_{ab}$ , such as  $g_{ab} + h_{ab}$ , which accounts consistently for the fluctuations of the stress tensor. The simplest equation is obtained by adding to the right hand side of equation (1), but written for the perturbed metric, the stochastic tensor just introduced. Subtraction of the semiclassical equation (1) leads to the following equation,

$$G_{ab}^{(1)}[g + h] = \kappa \langle \hat{T}_{ab}^{(1)}[g + h] \rangle_{ren} + \kappa \xi_{ab}[g], \quad (5)$$

where we recall that the background metric  $g_{ab}$  is assumed to be a solution of equation (1). As indicated by the superscript (1) this stochastic equation must be thought of as a linear equation for the metric perturbation  $h_{ab}$  which will behave consequently as a stochastic field tensor. Note that the tensor  $\xi_{ab}[g]$  is not a dynamical source, since it has been defined in the background metric. Note also that this source is divergenceless with respect to the metric, and it is thus consistent to write it on the right hand side of the Einstein equation. This equation is gauge invariant with respect to diffeomorphisms defined by any field on the background spacetime [43]. If we take the statistical average, equation (5) becomes just the semiclassical equation for the metric  $g_{ab} + \langle h_{ab} \rangle_s$  where the expectation value of  $\hat{T}_{ab}$  is taken in the perturbed spacetime. The quantum field now propagates in the spacetime described by the perturbed metric and thus it satisfies

$$(\nabla_{g+h}^2 - m^2 - \xi R[g + h]) \hat{\phi} = 0, \quad (6)$$

where the Ricci scalar is evaluated for the perturbed metric.

The stochastic equation (5) is known as the Einstein-Langevin equation. The equation predicts that the gravitational field has stochastic fluctuations over the background  $g_{ab}$ . It is linear in  $h_{ab}$ , thus its solutions can be written as follows,

$$h_{ab}(x) = h_{ab}^0(x) + \kappa \int d^4 x' \sqrt{-g(x')} G_{abcd}^{ret}(x, x') \xi^{cd}(x'), \quad (7)$$

where  $h_{ab}^0(x)$  is the solution of the homogeneous equation containing information on the initial conditions and  $G_{abcd}^{ret}(x, x')$  is the retarded propagator of equation (5) with vanishing initial conditions. From this we obtain the two-point correlations for the metric perturbations:

$$\langle h_{ab}(x) h_{cd}(y) \rangle_s = \langle h_{ab}^0(x) h_{cd}^0(y) \rangle_s + \kappa^2 \int d^4 x' d^4 y' \sqrt{g(x') g(y')} G_{abef}^{ret}(x, x') N^{efgh}(x', y') G_{cdgh}^{ret}(y, y'). \quad (8)$$

There are two different contributions to the two-point correlations. The first one is connected to the fluctuations of the initial state of the metric perturbations, we refer to them as *intrinsic fluctuations*. The second contribution is proportional to the noise kernel and is thus connected with the fluctuations of the quantum fields, we will refer to them as *induced fluctuations*. These two-point stochastic correlations are the most relevant physical observable, to find

them requires to know the noise kernel  $N_{abcd}(x, y)$ . Note that the noise kernel should be thought of as a distribution function, the limit of coincidence points has meaning only in the sense of distributions. Explicit expressions of this kernel in terms of the two point Wightman functions is given in [43], expressions based on point-splitting methods have also been given in [54, 55].

These stochastic correlations for the metric perturbations satisfy a very important property. In fact, it can be shown that they correspond exactly to the symmetrized two-point quantum metric correlations obtained in the large  $N$  expansion:

$$\frac{1}{2}\langle\{\hat{h}_{ab}(x), \hat{h}_{cd}(y)\}\rangle = \langle h_{ab}(x)h_{cd}(y)\rangle_s, \quad (9)$$

where  $\hat{h}_{ab}(x)$  mean the quantum operator corresponding to the metric perturbations. This result was implicitly obtained in the Minkowski background in ref. [46] where the two-point correlation in the stochastic context was computed for the linearized metric perturbations. This stochastic correlation exactly agrees with the symmetrized part of the graviton propagator computed by Tomboulis [56] in the quantum context of gravity interacting with  $N$  matter Fermion fields, where the graviton propagator is of order  $1/N$ . This result can be extended to an arbitrary background in the context of the large  $N$  expansion [48], a sketch of the proof with explicit details in the Minkowski background can be found in ref. [47]. This connection between the stochastic correlations and the quantum correlations was noted and studied in detail in the context of simpler open quantum systems [45]. It is thus clear that stochastic gravity goes beyond semiclassical gravity in the following sense. The semiclassical theory, which is based on the expectation value of the stress energy tensor, carries information on the field two-point correlations only, since  $\langle\hat{T}_{ab}\rangle$  is quadratic in the field operator  $\hat{\phi}$ . The stochastic theory on the other hand, is based on the noise kernel (3) which is quartic in the field operator. However, it does not carry information on the graviton-graviton interaction, which in the context of the large  $N$  expansion it gives diagrams of order  $1/N^2$ . This will be illustrated in section IV.

### A. Functional approach

To end this section we should mention that the Einstein-Langevin equation (5) may also be derived using the CTP functional method [42]. As remarked in the introduction the CTP functional was introduced by Schwinger [20, 21, 22] to compute expectation values. One just considers the interaction of the gravitational field  $g_{ab}$ , classical, with the field  $\phi$ , fully quantum. Then the effective action for the gravitational field is derived after integrating out the degrees of freedom of the quantum field, and the CTP influence action reduces basically to the Feynman and Vernon influence functional [36, 37] used in quantum open systems. Here the system is the gravitational field and the environment is the quantum field. The stochastic terms for the gravitational field are found by suitably interpreting some pure imaginary term which appear in the influence action. These terms are closely connected to Gell-Mann and Hartle decoherence functional [41] used to study decoherence and classicalization in open quantum systems. The net result is that the interaction with the environment induces fluctuations in the system dynamics. It is precisely the noise kernel introduced in (3) that accounts for this effect.

## IV. THE LARGE $N$ EXPANSION

The large  $N$  expansion has been successfully used in quantum chromodynamics to compute some non-perturbative results. This expansion re-sums and rearranges Feynman perturbative series including self-energies. For gravity interacting with  $N$  matter fields it shows that graviton loops are of higher order than matter loops. To illustrate the large  $N$  expansion let us consider the following toy model of gravity, which we will simplify as a scalar field  $h$ , interacting with a scalar field  $\phi$  described by the Lagrangian density

$$L = \frac{1}{\kappa} \int d^4x (\partial_a h \partial^a h + h(\partial h)^2 + \dots) - \int d^4x (\partial_a \phi \partial^a \phi + m^2 \phi^2) + \int d^4x (h(\partial \phi)^2 + \dots), \quad (10)$$

where, as previously  $\kappa = 8\pi G$ , and we have assumed that the interaction is linear in the (dimensionless) scalar gravitational field  $h$  and quadratic in the matter field  $\phi$  to simulate in a simplified way the coupling of the metric with the stress tensor of the matter fields. We have also included a self coupling graviton term of  $O(h^3)$  which also appears in perturbative gravity beyond the linear approximation.

We may now compute the dressed graviton propagator perturbatively as the following series of Feynman diagrams. The first diagram is just the free graviton propagator which is of  $O(\kappa)$ , as one can see from the kinetic term for the graviton in equation (10). The next diagram is one loop of matter with two external legs which are the graviton

propagators. This diagram has two vertices with one graviton propagator and two matter field propagators. Since the vertices and the matter propagators contribute with 1 and each graviton propagator contributes with a  $\kappa$  this diagram is of order  $O(\kappa^2)$ . The next diagram contains two loops of matter and three gravitons, and consequently it is of order  $O(\kappa^3)$ . There will also be terms with one graviton loop and two graviton propagators as external legs, with three graviton propagators at the two vertices due to the  $O(h^3)$  term in the action (10). Since there are four graviton propagators which carry a  $\kappa^4$  but two vertices which have  $\kappa^{-2}$  this diagram is of order  $O(\kappa^2)$ , like the term with one matter loop.

Let us now consider the large  $N$  expansion. We assume that the gravitational field is coupled with a large number of identical fields  $\phi_j$ ,  $j = 1, \dots, N$  which couple only with  $h$ . Next we re-scale the gravitational coupling in such a way that  $\bar{\kappa} = \kappa N$  is finite even when  $N$  goes to infinity. The Lagrangian density of this system is:

$$L = \frac{N}{\bar{\kappa}} \int d^4x (\partial_a h \partial^a h + h(\partial h)^2 + \dots) - \sum_j^N \int d^4x (\partial_a \phi_j \partial^a \phi_j + m^2 \phi_j^2) + \sum_j^N \int d^4x (h(\partial \phi_j)^2 + \dots). \quad (11)$$

Now an expansion in powers of  $1/N$  of the dressed graviton propagator is given by the following series of Feynman diagrams. The first diagram is the free graviton propagator which is now of order  $O(\bar{\kappa}/N)$  the following diagrams are  $N$  identical Feynman diagrams with one loop of matter and two graviton propagators as external legs, each diagram due to the two graviton propagators is of order  $O(\bar{\kappa}^2/N^2)$  but since there are  $N$  of them the sum can be represented by a single diagram with a loop of matter of weight  $N$ , and therefore this diagram is of order  $O(\bar{\kappa}^2/N)$ . This means that it is of the same order as the first diagram in an expansion in  $1/N$ . Then there are diagrams with two loops of matter and three graviton propagators, as before we can assign a weight of  $N$  to each loop and taking into account the three graviton propagators this diagram is of order  $O(\bar{\kappa}^3/N)$ , and so on. This means that to order  $1/N$  the dressed graviton propagator contains all the perturbative sums in powers of  $\bar{\kappa}$  of the matter loops.

Next, there is a diagram with one graviton loop and two graviton legs. Let us count the order of this diagram: it contains four graviton propagators and two vertices, the propagators contribute as  $(\bar{\kappa}/N)^4$  and the vertices as  $(N/\bar{\kappa})^2$ , thus this diagram is of  $O(\bar{\kappa}^2/N^2)$ . Therefore graviton loop contributes to higher order in the  $1/N$  expansion than matter loops. Similarly there are  $N$  diagrams with one loop of matter with an internal graviton propagator and two external graviton legs. Thus we have three graviton propagators and since there are  $N$  of them, their sum is of order  $O(\bar{\kappa}^3/N^2)$ . To summarize, we have that when  $N \rightarrow \infty$  there are no graviton propagators and gravity is classical, this is semiclassical gravity. To next to leading order,  $1/N$ , the graviton propagator includes all matter loop contributions, but no contributions from graviton loops and internal graviton propagators in matter loops. This is what stochastic gravity reproduces.

That stochastic gravity is connected to the large  $N$  expansion can be seen from the stochastic correlations of linear metric perturbations on the Minkowski background computed in ref. [46]. These correlations are in exact agreement with the imaginary part of the graviton propagator found by Tomboulis in the large  $N$  expansion for the quantum theory of gravity interacting with  $N$  Fermion fields [56]. This has been proved in detail in ref. [47] and extended to the general case [48].

## V. GRAVITATIONAL FLUCTUATIONS DURING INFLATION

An important application of stochastic gravity is the derivation of the cosmological perturbations generated during inflation [54]. Let us consider the Lagrangian density for an inflaton field  $\phi$  of mass  $m$

$$\mathcal{L}(\phi) = \frac{1}{2} \partial_a \phi \partial^a \phi + \frac{1}{2} m^2 \phi^2, \quad (12)$$

which is the basis of the simplest chaotic inflationary model [57]. The conditions for the existence of an inflationary period, which is characterized by an accelerated expansion of the spacetime, is that the value of the field averaged over a region with the typical size of the Hubble radius is higher than the Planck mass  $m_P$ . After the Planck era one expects, by Heisenberg uncertainty principle, that in a region of the size of the Planck size  $m_P^{-1}$  the energy density would be of the order of  $m_P^4$ . If the inflaton field starts in one of those regions at a value much larger than  $m_P$  then with a potential like that of eq. (12) one expects an inflaton mass  $m \ll m_P$  in which case the potential is very flat and the field rolls slowly towards the minimum producing a period of almost de Sitter expansion of that region. In order to solve the cosmological horizon and flatness problem more than 60 e-folds of expansion are needed, to achieve this the scalar field should begin with a value higher than  $3m_P$ . Moreover, we will see that the large scale anisotropies measured [58, 59, 60] restrict the inflaton mass to be of the order of  $10^{-6} m_P$ .

We want to study the metric perturbations produced by the stress tensor fluctuations of the inflaton field on the homogeneous background of a flat Friedmann-Robertson-Walker model, described by the cosmological scale factor

$a(\eta)$ , where  $\eta$  is the conformal time, which is driven by the homogeneous inflaton field  $\phi(\eta) = \langle \hat{\phi} \rangle$ . Thus we write the inflaton field in the following form

$$\hat{\phi} = \phi(\eta) + \hat{\varphi}(x), \quad (13)$$

where  $\hat{\varphi}(x)$  corresponds to a free massive quantum scalar field with zero expectation value on the homogeneous background metric:  $\langle \hat{\varphi} \rangle = 0$ . Restricting ourselves to scalar-type perturbations the perturbed metric  $\tilde{g}_{ab} = g_{ab} + h_{ab}$  can be written in the longitudinal gauge as,

$$ds^2 = a^2(\eta)[-(1 + 2\Phi(x))d\eta^2 + (1 - 2\Psi(x))\delta_{ij}dx^i dx^j], \quad (14)$$

where the metric perturbations  $\Phi(x)$  and  $\Psi(x)$  correspond to Bardeen's gauge invariant variables [61, 62].

The Einstein-Langevin equation is gauge invariant, and thus we can work in a desired gauge and then extract the gauge invariant quantities. The Einstein-Langevin equation (5) reads now:

$$G_{ab}^{(1)}[h] - \kappa \langle \hat{T}_{ab}^{(1)}[h] \rangle = \kappa \xi_{ab}[g], \quad (15)$$

where  $g_{ab}$  satisfies the semiclassical Einstein equations; assuming slow roll this background metric is an almost de Sitter metric. The superscript (1) refers to functions linear in the metric perturbation  $h_{ab}$ . The stress tensor operator  $\hat{T}_{ab}$  for the minimally coupled inflaton field in the perturbed metric  $\tilde{g}_{ab}$  is:

$$\hat{T}_{ab} = \tilde{\nabla}_a \hat{\phi} \tilde{\nabla}_b \hat{\phi} + \frac{1}{2} \tilde{g}_{ab} (\tilde{\nabla}_c \hat{\phi} \tilde{\nabla}^c \hat{\phi} + m^2 \hat{\phi}^2). \quad (16)$$

Now, using the decomposition of the scalar field into its homogeneous and inhomogeneous part, see eq. (13), and the metric  $\tilde{g}_{ab}$  into its homogeneous background  $g_{ab}$  and its perturbation  $h_{ab}$ , the renormalized expectation value for the stress tensor can be written as

$$\langle \hat{T}_{ab}[\tilde{g}] \rangle = \langle \hat{T}[\tilde{g}] \rangle_{\phi\phi} + \langle \hat{T}_{ab}[\tilde{g}] \rangle_{\phi\varphi} + \langle \hat{T}_{ab}[\tilde{g}] \rangle_{\varphi\varphi}, \quad (17)$$

where only the homogeneous solution for the scalar field contributes to the first term. The second term is proportional to  $\langle \hat{\varphi}[\tilde{g}] \rangle$  which is not zero because the field dynamics is considered on the perturbed spacetime, i.e. this term includes the coupling of the field with  $h_{ab}$ , as seen in equation (6). The last term corresponds to the expectation value to the stress tensor for a free scalar field on the spacetime of the perturbed metric.

We can now compute the noise kernel  $N_{abcd}(x, y)$  defined in equation (3), which after using the previous decomposition may be written as

$$\langle \{\hat{t}_{ab}, \hat{t}_{cd}\} \rangle[g] = \langle \{\hat{t}_{ab}, \hat{t}_{cd}\} \rangle_{\phi\phi}[g] + \langle \{\hat{t}_{ab}, \hat{t}_{cd}\} \rangle_{\varphi\varphi}[g], \quad (18)$$

where we have used the fact that  $\langle \hat{\varphi} \rangle = 0 = \langle \hat{\varphi} \hat{\phi} \hat{\varphi} \rangle$  for Gaussian states on the background geometry. We have considered the vacuum state to be the Bunch-Davies, or Euclidean, vacuum which is preferred in the de Sitter background, and this state is Gaussian. In the above equation the first term is quadratic in  $\hat{\varphi}$  whereas the second one is quartic, both contributions to the noise kernel are separately conserved since both  $\phi(\eta)$  and  $\hat{\varphi}$  satisfy the Klein-Gordon field equations on the background spacetime. Consequently, the two terms can be considered separately. On the other hand if one treats  $\hat{\varphi}$  as a small perturbation the second term in (18) is of lower order than the first and may be neglected consistently, this corresponds to neglecting the fluctuations associated with the last term in equation (17). This approximation is equivalent to keep only linear terms in the inflaton perturbations. Stress tensor fluctuations due to a term like the last term of (17) were considered in ref. [63].

We can now write down the Einstein-Langevin equations (15). It is easy to check that the *space-space* components coming from the stress tensor expectation value terms and the stochastic tensor are diagonal, i.e.  $\langle \hat{T}_{ij} \rangle = 0 = \xi_{ij}$  for  $i \neq j$ . This, in turn, implies that the two functions characterizing the scalar metric perturbations are equal:  $\Phi = \Psi$  in agreement with ref. [62]. The equation for  $\Phi$  can be obtained from the 0i-component of the Einstein-Langevin equation, which in Fourier space reads

$$2ik_i(H\Phi_k + \Phi'_k) = \frac{8\pi}{m_P^2} \xi_{k0i}, \quad (19)$$

where  $k_i$  is the comoving momentum component associated to the comoving coordinate  $x^i$ . Here primes denote derivatives with respect to the conformal time  $\eta$  and  $H = a'/a$ . A non-local term of dissipative character which comes from the second term in (17) should also appear on the left hand side of equation (19), but we have ignored this term for simplicity (if one includes this non-local term it is then more convenient to write an equation combining the

other equations which is free of non-local terms; but the results are not substantially altered). To solve this equation, whose left hand side comes from the linearized Einstein tensor for the perturbed metric [62], we need the retarded propagator for the gravitational potential  $\Phi_k$ ,

$$G_k(\eta, \eta') = -i \frac{4\pi}{k_i m_P^2} \left( \theta(\eta - \eta') \frac{a(\eta')}{a(\eta)} + f(\eta, \eta') \right), \quad (20)$$

where  $f$  is a homogeneous solution of (19) related to the initial conditions chosen. For instance, if we take  $f(\eta, \eta') = -\theta(\eta_0 - \eta') a(\eta')/a(\eta)$  the solution would correspond to “turning on” the stochastic source at  $\eta_0$ .

The correlation function for the metric perturbations is now given by

$$\langle \Phi_k(\eta) \Phi_{k'}(\eta') \rangle_s = (2\pi)^2 \delta(\vec{k} + \vec{k}') \int^\eta d\eta_1 \int^{\eta'} d\eta_2 G_k(\eta, \eta_1) G_{k'}(\eta', \eta_2) \langle \xi_{k \ 0i}(\eta_1) \xi_{k' \ 0i}(\eta_2) \rangle_s. \quad (21)$$

The correlation function for the stochastic source, which is connected to the stress tensor fluctuations through the noise kernel is given by,

$$\langle \xi_{k \ 0i}(\eta_1) \xi_{-k \ 0i}(\eta_2) \rangle_s = \frac{1}{2} \langle \{ \hat{t}_{0i}^k(\eta_1), \hat{t}_{0i}^{-k}(\eta_2) \} \rangle_{\phi\varphi} = \frac{1}{2} k_i k_i \phi'(\eta_1) \phi'(\eta_2) G_k^{(1)}(\eta_1, \eta_2), \quad (22)$$

where  $G_k^{(1)}(\eta_1, \eta_2) = \langle \{ \hat{\varphi}_k(\eta_1), \hat{\varphi}_{-k}(\eta_2) \} \rangle$  is the  $k$ -mode Hadamard function for a free minimally coupled scalar field which is in the Euclidean vacuum on the de Sitter background.

It is useful to compute the Hadamard function for a massless field and consider a perturbative expansion in terms of the dimensionless parameter  $m/m_P$ . Thus we consider  $\bar{G}_k^{(1)}(\eta_1, \eta_2) = a(\eta_1) a(\eta_2) G_k^{(1)}(\eta_1, \eta_2) = \langle 0 | \{ \hat{y}_k(\eta_1), \hat{y}_{-k}(\eta_2) \} | 0 \rangle = 2\mathcal{R}(u_k(\eta_1) u_k^*(\eta_2))$  with  $\hat{y}_k(\eta) = a(\eta) \hat{\varphi}_k(\eta) = \hat{a}_k u_k(\eta) + \hat{a}_{-k}^\dagger u_{-k}^*(\eta)$  and where  $u_k = (2k)^{-1/2} e^{ik\eta} (1 - i/\eta)$  are the positive frequency  $k$ -mode for a massless minimally coupled scalar field on a de Sitter background, which define the Euclidean vacuum state:  $\hat{a}_k | 0 \rangle = 0$  [1].

The background geometry, however, is not exactly that of de Sitter spacetime, for which  $a(\eta) = -(H\eta)^{-1}$  with  $-\infty < \eta < 0$ . One can expand in terms of the “slow-roll” parameters and assume that to first order  $\dot{\phi}(t) \simeq m_P^2(m/m_P)$ , where  $t$  is the physical time. The correlation function for the metric perturbation (21) is then computed, see ref. [54] for details. The final result, however, is very weakly dependent on the initial conditions as one may understand from the fact that the accelerated expansion of de quasi-de Sitter spacetime during inflation erases the information about the initial conditions. Thus one may take the initial time to be  $\eta_0 = -\infty$  and obtain to lowest order in  $m/m_P$  the expression

$$\langle \Phi_k(\eta) \Phi_{k'}(\eta') \rangle_s \simeq 8\pi^2 \left( \frac{m}{m_P} \right)^2 k^{-3} (2\pi)^3 \delta(\vec{k} + \vec{k}') \cos k(\eta - \eta'). \quad (23)$$

From this result two main conclusions are derived. First, the prediction of an almost Harrison-Zel’dovich scale-invariant spectrum for large scales, i.e. small values of  $k$ . Second, since the correlation function is of order of  $(m/m_P)^2$  a severe bound to the mass  $m$  is imposed by the gravitational fluctuations derived from the small values of the Cosmic Microwave Background (CMB) anisotropies detected by COBE [58] and WMAP [59, 60]. This bound is of the order of  $(m/m_P) \sim 10^{-6}$  [58, 62]. One possible advantage of the Einstein-Langevin approach to the gravitational fluctuations in inflaton over the approach based on the quantization of the linear perturbations of both the metric and the inflaton field [62], is that an exact treatment of the inflaton quantum fluctuations is possible. On the other hand although the gravitational fluctuations are here assumed to be classical, the correlation functions obtained correspond to the expectation values of the quantum metric perturbations, as we have remarked in the previous section [45, 48].

## VI. FLUCTUATIONS NEAR BLACK HOLE HORIZONS

Another interesting application of stochastic gravity is found in the context of black hole physics, in particular the stress tensor fluctuations near the black hole horizon may induce fluctuations in the horizon area. The relevance of these fluctuations in Hawking radiation needs to be understood [64]. Some preliminary investigations seem to indicate that the fluctuations of the black hole horizon are always small and that the Hawking result should not be substantially different [65, 66], however, some other results by Bekenstein [67] seem to point in the opposite direction suggesting that the fluctuations of the black hole horizon may be significant in the long run. The contribution of the horizon fluctuations to the black hole entropy [68, 69] is another interesting issue that may deserve some attention in the present context.



To clarify this situation Hu and Roura [70] have analyzed this back-reaction problem in the stochastic gravity framework. Due to technical difficulties this is still work in progress so I will only summarize very briefly the main results. As shown by Hawking a black hole formed by spherical collapse emits thermal radiation with a temperature  $T = m_P^2/8\pi M$ , where  $M$  is the mass of the black hole. This calculation was made under the assumption of quantum field theory in a curved background. That is, assuming that the black hole has a fixed mass much larger than the Planck mass  $m_P$ , so that one can safely ignore quantum gravity effects, and that the black hole exterior can be described by the Schwarzschild metric. But, energy conservation arguments indicate that as the black hole emits radiation it will loss mass and evaporate. A precise calculation of the evaporation process requires the use of the semiclassical Einstein equations which describes the back-reaction of the quantum matter fields on the gravitational field in a self-consistent way. An exact self-consistent calculation of the evaporation process is by no means easy and has not been performed. Note that the black hole exterior is not vacuum any more, as the expectation value of the stress tensor  $\langle \hat{T}_{ab} \rangle_{ren}$  is not zero, and is not described exactly by the Schwarzschild metric. Thus, even the radiation process needs to be reviewed. Furthermore even in the Schwarzschild background an exact analytic expression to describe that expectation value in the Unruh vacuum, which is the natural quantum state that describes the initial vacuum in a black hole formed by gravitational collapse, is not known.

Fortunately, for large black holes the evaporation process is slow and a quasi-adiabatic approximation can be used to solve Einstein semiclassical equations. In this approximation one can assume that the black hole exterior is described by a Schwarzschild metric with a mass  $M$  which is the mass that the black hole has at that time. A suitable parameter to use in this approximation is the luminosity at a given time  $L_H = B/M^2$ , where  $B$  is a constant which depends on the number of fields considered, the spins of these fields and the grey body factor. It has been estimated by Page [71] to be of the order of  $10^{-4}$ . Here, and in the rest of this section, we use units in which  $m_P = 1$ . The quasi-adiabatic approximation holds as long as  $L_H \ll 1$ . Black hole evaporation in the adiabatic approximation was described by Bardeen [72] and Massar [73] for spherically symmetric black holes. Let us summarize this calculation. A spherically symmetric metric can always be written as

$$ds^2 = -e^{\psi(v,r)} \left[ 1 - \frac{2m(v,r)}{r} \right] dv^2 + 2e^{\psi(v,r)} dv dr + r^2(d\theta^2 + \sin^2 \theta d\varphi^2). \quad (24)$$

There is an apparent horizon where the expansion of the outgoing null geodesics vanish, that is where  $dr_a/dv = 0$ , which leads to  $r_a(v) = 2m(v, r_a(v)) \equiv 2M(v)$ . Thus we have defined  $2M(v)$  as the apparent horizon, when  $M$  is constant this corresponds to the event horizon and  $M$  is the black hole mass. Thus in the adiabatic approximation we may consider that  $M(v)$  is the black hole mass at the “advanced” time  $v$ . The semiclassical Einstein equations become in the above coordinates,

$$\frac{\partial m}{\partial v} = 4\pi r^2 \langle T_v^r \rangle, \quad \frac{\partial m}{\partial r} = -4\pi r^2 \langle T_v^v \rangle, \quad \frac{\partial \psi}{\partial v} = 4\pi r^2 \langle T_{rr} \rangle. \quad (25)$$

We do not have an analytic expression for the expectation value of the stress tensor even in the Schwarzschild spacetime. However, at large radii it corresponds to a thermal flux of radiation and we may write  $\langle T_v^r \rangle = L_H/(4\pi r^2)$ . Then one can use the stress tensor conservation equation to relate components on the horizon and far from it,

$$\frac{\partial(r^2 \langle T_v^r \rangle)}{\partial r} + r^2 \frac{\partial \langle T_v^v \rangle}{\partial v} = 0. \quad (26)$$

Using this equation one may relate the positive energy flux radiated away from the horizon and the negative energy flux crossing the horizon. Taking this relation into account and the quasi-adiabatic approximation one finally gets, from the first of equations (25) at the horizon, the equation for the evolution of the apparent horizon:

$$\frac{dM}{dv} = -\frac{B}{M^2}, \quad (27)$$

which gives the evaporation rate, as one would expect from energy conservation considerations.

Now the Einstein-Langevin equation may be used to study the metric fluctuations near the black hole horizon of the evaporating black hole. A full self-consistent computation is technically involved, in particular the computation of the noise kernel is very complicated, but Roura and Hu [70] where able to give some reasonable estimates of the event horizon fluctuations in the evaporating process. They concentrate on the spherically symmetric fluctuations by projecting the Einstein-Langevin equation on the spherical sector. The Einstein-Langevin equation for the perturbation of  $m(v,r)$ ,  $\delta m(v,r)$ , can be written in the quasi-adiabatic approximation as

$$\frac{\partial(\delta m)}{\partial v} = \frac{2B}{m^3} \delta m + 4\pi r^2 \xi_v^r + O(L_H^2), \quad (28)$$

where  $\xi_v^r$  is the Gaussian stochastic component defined through the noise kernel components  $\frac{1}{2}\langle\{\hat{t}_v^r\hat{t}_v^r\}\rangle$ . Here one is interested in the fluctuations near the apparent horizon. The noise kernel there has not yet been computed, however, far from the horizon it has been estimated by Wu and Ford [66]. They found a correlation time for the fluctuations of the stress tensor of the order of  $M$  and smearing the 2-point functions over this correlation time they found fluctuations of the order of  $1/M^4$ . From this Hu and Roura [70] deduced that

$$\langle\xi(v)\xi(v')\rangle_s \sim \frac{1}{M^3(v)}\delta(v-v'), \quad (29)$$

which for times larger than the correlation time reproduces the Wu-Ford result [66]. Here  $\xi(v) \equiv (4\pi r^2 \xi_v^r)[v, r \sim 6M(v)]$ , relatively far from the horizon.

One then assumes that as a consequence of the divergenceless property of the stochastic tensor which leads to an equation like (25), but where  $\xi_a^b$  replaces  $\langle T_a^b \rangle$ , one may be able to connect in a simple way the value of the stochastic source near the horizon with its value at large radii. Then, under the assumption that equation (29) is still valid at the horizon Hu and Roura [70] found that if  $M_0$  is the initial mass of the black hole the fluctuations of  $M(v)$  are,

$$\delta M \sim \left(\frac{M_0}{M}\right)^2, \quad (30)$$

which imply that the fluctuations grow in time and they become of the order of the mean value  $\delta M(v) \sim M(v)$  when  $M(v) \sim M_0^{2/3}$ . From the evaporation rate equation (27) the evaporation time of the black hole is of order of  $M_0^3$  and thus it reaches the mass  $M_0^{2/3}$  after a significant fraction of the evaporation time. For instance for a black hole with an initial mass of the order the solar mass  $M_0 = 1 M_\odot$  the fluctuations become comparable to its mean value when it reaches a Schwarzschild radius of the order of  $r_S \sim 10$  nm and thus its mass is still much larger than the Planck mass and the semiclassical approximation should hold.

Therefore, according to the previous estimation the fluctuations grow and accumulate in time, the source of this accumulation is the non local term in the Einstein-Langevin equation, which originates the first term on the right hand side of equation (28). This result agrees with the estimation by Bekenstein [67] who also found this long time enhancement of the fluctuations. It differs, however, from the estimations by Wu and Ford [66] who neglected the non local term. If true, this result seems to point to the breaking of the semiclassical approximation well before the black hole reaches the Planck mass and a re-examination of the evaporation process may be needed. At the moment we have to take this result with a grain of salt, as some of the approximations used connecting the stochastic tensor near the horizon and far from it are not totally correct [70]. But it seems clear that more work is needed, in particular a better approximation for the noise kernel near the black hole horizon even within the semi-adiabatic approximation is needed. The noise kernel must be treated as a distribution, it is singular in the coincidence limit and for null separated points, but it is finite if properly smeared with smooth functions suitably integrated in time as well as in space.

## VII. SUMMARY AND OUTLOOK

We have reviewed the semiclassical theory of gravity as the theory of the interaction of classical gravity with quantum matter fields. The most important equations in this theory are the semiclassical Einstein equations (1) which describe the back-reaction of the gravitational fluctuations in its interaction with the quantum fields. We noticed that the theory may seriously fail when the fluctuations on the stress tensor of the quantum fields are significant. We have then sought an axiomatic approach by which the semiclassical equations can be corrected in order to take into account those fluctuations. These equations turn out to be uniquely defined and are the Einstein-Langevin equations (5) which are linear in the metric perturbations  $h_{ab}$  over the semiclassical background. These equations predict stochastic fluctuations in the metric perturbations induced by the stress tensor fluctuations described by the noise kernel (3). We have also noted that the stochastic correlations of the metric perturbations predicted by the Einstein-Langevin equations reproduce the quantum metric correlations of the quantum theory of gravity interacting with  $N$  matter fields, in the large  $N$  expansion.

We have finally used the stochastic theory in the inflationary cosmological context. We have computed the two-point correlation functions of the metric fluctuations during a quasi-de Sitter expansion induced by the stress tensor fluctuations of the inflaton field. The results are in agreement with other approaches to the same problem [62], an approximate Harrison-Zel'dovich spectrum is predicted. We noticed that in our approach the quantum fields and the gravitational fields are treated separately, and this may have some advantages to go one step further and consider the quantum field fully, not just to linear order. We have also considered a second application in black hole

physics. We have argued that for a large evaporating black hole, fluctuations can accumulate over time and become significant before reaching Planck scales. But more work is needed to confirm this calculation and to explore its possible consequences.

Other applications of stochastic semiclassical gravity to semiclassical cosmology have been performed [74], some including thermal fields [44, 75]. It has been shown that noise produced by a quantum field on the cosmological scale factor of an isotropic closed Friedmann-Robertson-Walker, in the presence of a cosmological constant, may take the scale factor from a region where it is nearly zero to a region where it describes a de Sitter inflationary era [76]. Thus jumping over the barrier by activation, this is the semiclassical analogue of the tunneling from nothing in quantum cosmology [77, 78, 79] and gives yet another mechanism to produce inflation. Finally, stochastic gravity has also been used to formulate a criteria for the validity and stability of semiclassical gravity [47]. In particular it has been shown, in this context, that flat spacetime as a background solution of semiclassical gravity is stable.

### Acknowledgments

I am very grateful to the organizers of the *XXIX Spanish Relativity Meeting, ERE-2006* for giving me the opportunity to participate at the conference on “*Einstein’s legacy: from the theoretical paradise to astrophysical observation*”, and for their kind and generous hospitality. I am also grateful to Rosario Martín and Albert Roura for their essential contribution to the work described here, and to Daniel Arteaga, Esteban Calzetta, Antonio Campos and Bei-Lok Hu for many discussions during the last years on this topic. I also thank Albert Roura for a critical reading of the manuscript and many useful suggestions. This work has been partially supported by the Research projects MEC FPA-2004-04582 and DURSI 2005SGR00082.

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